Modulation Distortion in Loudspeakers*

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When comparing a loudspeaker with direct radiator bass system to one with horn loaded bass, the subjective judgment is that the one with the horn loaded bass is "cleaner". The difference in listening quality appears to be due to modulation distortion. The mathematical analysis of modulation distortion is reviewed and spectrum analyzer measurements are described which have been correlated with listening tests. The spectrum analyses corroborate the mathematical analysis and the listening tests offer a subjective evaluation. It is concluded that frequency modulation in loudspeakers accounts in large measure for the masking of "inner voices". Reduction of diaphragm excursions at low frequencies reduces FM distortion. Horn loading, properly applied, offers greatest reduction, while simultaneously improving bass power output capability.

INTRODUCTION The term *distortion* is defined herein as the generation of frequencies not originally present. Thus it is distinguished from frequency response errors. *Harmonic distortion* is the introduction of harmonics of the original frequencies and is not objectionable as such even at high amounts, because music consists largely of harmonics. Modulation distortion produces new frequencies which are inharmonically related to the original sounds and is therefore detectible and objectionable in much smaller amounts.

Amplitude modulation distortion derives from any nonlinearity which would produce harmonic distortion and appears as the inharmonic sum and difference frequencies or sidebands when two or more frequencies are mixed. The subject of amplitude modulation is covered in texts on radio engineering [1].

Great effort has been expended to reduce harmonic distortion in loudspeakers. Since harmonic distortion arises from the same causes as amplitude modulation (AM) distortion, it may tentatively be concluded that AM distortion in better loudspeakers is relatively small. This is not to dismiss AM distortion; the main effort in this paper is directed at total modulation distortion.

Frequency modulation distortion arises in a loudspeaker when diaphragm motion at some low frequency produces frequency shifts of some higher frequency due to the Doppler effect. The effect is similar to flutter in a defective tape player when the tape velocity is not constant within tolerable limits.

Experiments with an eccentric capstan on a tape machine indicate that flutter (frequency deviation) amounting to 0.35% is irritating if the modulating frequency is as high as 20 or 40 Hz [2].

Vast effort on the part of tape recorder manufacturers to attain wow and flutter levels below 0.1% deviation indicate that this problem is recognized. Yet much higher values are typically observed in loudspeakers.

The Doppler effect or the principle of alteration of pitch dates back to 1842 [3]. The familiar example is that of a moving vehicle, such as a locomotive, blowing a whistle. As the vehicle passes a listener the pitch of the whistle diminishes. The wavelengths observed are changed proportionally to the ratio of the velocity of the moving source to the velocity of sound.

A loudspeaker diaphragm vibrating at two frequencies, for example at 32 and 1000 Hz, would give rise to FM distortion. As the cone, vibrating at 1000 Hz, is moved toward and away from the observer, at the 32 Hz frequency there would arise a deviation from 1000 Hz dependent on the velocity of the diaphragm at the 32 Hz frequency.

If the velocity of the cone at the lower frequency were 0.5% of the velocity of sound, the frequency deviation of the higher frequency would be 0.5%.

Total modulation distortion is the combination of the amplitude and frequency modulation.

INITIAL OBSERVATIONS

Two loudspeakers, one with direct radiator bass and the other with horn-loaded bass, have been compared in listening tests. Nineteen out of 20 listeners judged that the one with the horn-loaded bass is "cleaner". Both loudspeakers were by the same manufacturer. Both had similar horn-loaded midrange and treble system with identical midrange and treble drive motors. The direct radiator unit offered a more extended bass range, with cutoff about 30 Hz compared to the horn-loaded bass cutoff of about 45 Hz. Thus, if the listener preference were predicated on frequency response the leaning should have been toward the direct radiator unit, but the opposite choice was indicated. The words "transparent", "better resolution", and "clarity of the inner voices" were applied to the speaker with the horn-loaded bass.

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Single-frequency distortion tests indicated that both loudspeakers generate negligible harmonic distortion at the power levels used in the listening tests. Both showed accurate power linearity up to 120 dB SPL measured at 4 ft, as indicated by parallelism of frequency response curves run at 0.1, 1.0 and 10 W input. Hysteresis effects, if any, were below the resolving power of the measuring equipment used.

By elimination it appears that the cause of the difference in listening quality must be modulation distortion.

EARLY WORK ON FM DISTORTION

Frequency modulation distortion has long been recognized as a property of loudspeakers [4]. When two frequencies are radiated by the same diaphragm the higher frequency f_2 will be frequency-modulated by the lower frequency f_1 because of the Doppler shift due to the motion at the lower frequency. This frequency shift of the upper frequency will be proportional to the ratio of the diaphragm velocity at the lower frequency to the velocity of sound.

Let A_1 be the peak amplitude of motion due to f_1, then

$$x = A_1 \sin \omega_1 t \qquad (1)$$

$$v = dx/dt = A_1 \omega_1 \cos \omega_1 t \qquad (2)$$

or the peak velocity

$$V = A_1 \omega_1.$$

The shift or deviation of a higher frequency will be proportional to V/c or $\Delta f_2 = (V/c) f_2$.

Writing c = 13,500 ips (velocity of sound),

$$\Delta f_2 = f_2 A_1 \omega_1 / 13{,}500. \qquad (3)$$

For example, A_1 = ⅛ in., f_1 = 28 Hz, ω_1 = 176, V = 176/8 = 22 ips, V/c = 22/13,500 = 0.0016.

If f_2 = 600 Hz, then Δf_2 = 0.0016 × 600 = 1 Hz. In this example, therefore, the frequency deviation would be one Hz and the frequency f_2 would flutter between 599 and 601 Hz.

The amount of frequency shift, flutter, or deviation is easy to determine by simply measuring the diaphragm excursion at the lowest frequency, from which, knowing the frequency, the modulating velocity may be computed. The frequency modulation shift is simply this velocity divided by the velocity of sound multiplied by the modulated frequency.

Beers and Belar [4] derive a different measure for frequency modulation distortion, consisting of the effective amplitude of the sidebands.

Beers and Belar give

$$d = 0.033\, A_1 f_2 \qquad (4)$$

where d = distortion factor (total rms value of sidebands as percent of amplitude of f_2), A_1 = amplitude of motion in inches at the lower frequency f_1, and f_2= the higher or modulated frequency.

For the example given, where the cone excursion was ⅛ in. (amplitude A_1 = 0.12 in.) and the upper frequency f_2 = 600 Hz, d = 0.033 × 0.12 × 600 = approximately 2.4%. From the principles of frequency modulation, it may be seen that the sidebands consist of the sum and difference frequencies between the upper frequency f_2 and the modulating frequency f_1, as well as their higher orders; thus the sideband frequencies would be

$$f_2 \pm f_1$$
$$f_2 \pm 2f_1, \text{ etc.}$$

Note that the *amplitude* A_1 is a factor in determining the amplitude of the sidebands; the *velocity* V_1 determines the fraction of frequency shift of the modulated frequency.

FM ANALYSIS

The equation of frequency modulation is given by Terman [5]; in slightly rearranged form it is

$$e = E \sin\left[\omega_2 t + (\Delta\omega_2/\omega_1)\sin\omega_1 t\right], \qquad (5)$$

where E is the amplitude of the "carrier" or higher frequency; $\omega_2 = 2\pi f_2$, with f_2 the higher or modulated frequency; $\omega_1 = 2\pi f_1$, with f_1 the lower or modulating frequency; and $\Delta\omega_2$ is the maximum deviation of the instantaneous frequency f_2.

Let

$$E = 1,\ m = \frac{\Delta\omega_2}{\omega_1} = \frac{(V/c)\,\omega_2}{\omega_1} = \frac{A_1\omega_1}{c}\cdot\frac{\omega_2}{\omega_1} = \frac{A_1}{c}\omega_2, \qquad (6)$$

$$e = \sin(\omega_2 t + m \sin\omega_1 t) \qquad (7)$$

$$\begin{aligned} &= J_0 m \sin\omega_2 t \\ &\quad + J_1 m\left[\sin(\omega_2+\omega_1)t - \sin(\omega_2-\omega_1)t\right] \\ &\quad + J_2 m\left[\sin(\omega_2+2\omega_1)t + \sin(\omega_2-2\omega_1)t\right] \\ &\quad + \text{etc.} \end{aligned} \qquad (8)$$

Expressed as side band amplitudes [6], for $m << 1$,

$$\begin{aligned} e_0 &= J_0 m \cong 1 \\ e_1 &= J_1 m \cong m/2 \\ e_2 &= J_2 m \cong m^2/8, \text{ etc.} \end{aligned} \qquad (9)$$

Using c = 13,500 ips,

$$\begin{aligned} e_0 &= 1 \\ e_1 &\cong 0.00023\, A_1 f_2 \\ e_2 &\cong 2.7 \times 10^{-8} (A_1 f_2)^2 \text{ etc.,} \end{aligned} \qquad (10)$$

where e_1 is the magnitude of *each* of the 2 sidebands of first order; e_2 is magnitude of each second order sideband, etc.

Equations (10) and (4) can be compared. Consider the amplitude of one first-order sideband frequency

$$e_1 = 0.000233\, A_1 f_2 \qquad (10)$$

where A_1 is the amplitude of motion due to f_1; if the second-order sidebands are small enough to be ignored, the effective value of both first-order sideband frequencies would be

$$\begin{aligned} E_1 &= 0.000233 \sqrt{2}\, A_1 f_2 \\ &= 0.00033\, A_1 f_2 \end{aligned}$$

which agrees with Beers and Belar's Eq. 4 where their coefficient of 0.033 was expressed in percent. Numerical example:

A_1 = ⅛ in.
e_1 = 0.017 (approx. −35 dB)
e_2 = 0.00013 (approx. −78 dB).

Two sidebands e_1 of 0.017 would represent an rms amplitude of

$$0.017 \times \sqrt{2} = 0.024,$$

which agrees with the 2.4% obtained using Beers-Belar's method.

Equations (8) through (11) state that the sidebands contain frequencies of $f_1 \pm f_2$, $f_1 \pm 2f_2$, etc. Equation (3) suggests the sidebands are limited to the deviation. Terman [5] offers the explanation that while the frequency is shifting the wave becomes lopsided, giving rise to other sinusoidal components.

Qualitatively it is hard to reconcile a one Hz deviation with a 28 Hz sideband component but these sideband frequencies become credible when viewed on the spectrum analyzer screen.

Beers and Belar draw several conclusions: 1. since the distortion factor d increases with f_2 the effect will be more severe in "high fidelity" applications, so that two loudspeakers should be used, one to radiate the low frequencies and the other for highs. (Recall that in 1943 a so-called two-way speaker was used mainly in theaters.) 2. It is difficult to evaluate FM distortion with listening tests because it cannot be readily isolated from other forms of distortion, and 3. FM distortion is probably masked by other forms of distortion.

When this was written in 1943, with AM radio and shellac disk records as the main media, it was probably true that FM distortion in loudspeakers was masked by other forms of distortion and noise. Now, with amplifiers exhibiting 0.05% distortion, FM radio eliminating some forms of distortion exhibited by AM and minimizing others, and with tape offering 55 dB or better S/N ratio, it is suspected that FM distortion in loudspeakers may be the major remaining defect in otherwise high-quality audio systems.

TOTAL MODULATION DISTORTION

Amplitude modulation gives rise to the same families of distortion sidebands ($f_2 \pm f_1$, etc.) as does frequency modulation. Using a spectrum analyzer, the indicated amplitude of each sideband component is the effective sum of the two kinds of distortion. Sometimes two sideband frequencies $f_1 + f_2$ and $f_1 - f_2$ are shown with different amplitudes. This is possibly due to a phase difference between the amplitude and frequency modulation components so that the effective sum of one side component differs from that of the other. The effective sum of both components will still represent the effective value of that particular order of modulation distortion. The effective sum of all sideband components will be the "total modulation distortion" for that particular combination of inputs.

No obvious way of separating amplitude and frequency modulation distortion has so far been described. But since the two kinds produce the same type of sidebands, the total modulation distortion is what causes listener distress. Fortunately the spectrum analyzer can be used to measure this total modulation distortion.

EXPERIMENTAL RESULTS

The experimental studies were performed with the following equipment:

Two oscillators, Hewlett-Packard HO2-207A GR 1310A;

Dual power amplifier, solid state "one of a kind", arranged to simplify two oscillators separately, feeding combined output to loudspeaker. Measurements indicate negligible distortion under conditions imposed;

Various control boxes;

B & K half-inch microphone;

Tektronix 564 storage scope with 3L5 spectrum analyzer and 2B67 time base.

Bass Loudspeakers

This must necessarily be merely a "progress report". It is doubtful if the work will ever be finished.

Table I compares the performance of four bass loudspeakers. Frequencies of 50 Hz and 300 Hz were used, and output at each frequency adjusted to give the stated SPL. (See Fig. 1).

The table confirms a rule which has long been known but not rigorously proved, namely that the higher the efficiency the lower the total modulation distortion. Another fact (not previously realized) is that the low-efficiency loudspeakers exhibit much higher total distortion than the computed FM distortion; highly efficient loudspeakers exhibit a total distortion only slightly greater than the computed FM distortion. The difference must be AM distortion. In the case of the high-efficiency horn loudspeaker, the individual sidebands were about 45 dB down from the 300 Hz signal. Combined distortion of oscillator, amplifier and analyzer could account for some of the total.

Let it be repeated that this work will never be finished. Results so far indicate a trend, and it is tentatively suggested that the last column in Table I be called "mud index".

Good loudspeakers are referred to as "clean" or "transparent", and poor speakers as "dirty" or "muddy". The magnitude of the modulation distortion is directly correlatable to the quality of "muddiness". Thus the term "mud index" may need no defense.

The main qualities of a loudspeaker have been considered to be: 1. Power output capacity, at 2. a corresponding distortion level, 3. polar response, and 4. frequency response.

If modulation distortion is quantitatively specified under 2. above, then it appears that loudspeakers can indeed be described as accurately as amplifiers.

Table I. Loudspeaker performance: bass loudspeakers.

	Output Sound Pressure Level (SPL) dB at 2 Feet	% Total Modulation Distortion (Effective Sideband Amplitude)
Large well-designed horn	100	0.7
15 in. driver in ported box 6.5 ft^3	95	2.2
10 in. driver in sealed box 1.5 ft^3	95	6.8
7 in. driver in sealed box 1.0 ft^3	90	14.0

Midrange Loudspeakers

Again, this is only a progress report.

Tests were run on a high-quality midrange horn-type loudspeaker, a reflexed horn midrange and an 8 in. direct radiator (see Table II and Fig. 2).

Table II. Loudspeaker performance: midrange loudspeakers.

	Output SPL, dB at 2 ft	Modulation Distortion (Effective Sidebands)
Straight axis horn	100	less than 1%
Reflexed horn	100	approximately 5%
8 in. direct radiator	90	approximately 10%

As in bass loudspeakers, the distortion was approximately inversely proportional to efficiency. In the case of the reflexed horn the peak/trough ratio in the operating spectrum was about 24 dB; by choosing one frequency in the trough it was found that SPL values of only 90 dB could be obtained except under gross distortion. At 90 dB the total modulation distortion rose to 10%.

Experiments so far tend to indicate that plastic diaphragms in the compression drivers for horns exhibit slightly lower total modulation distortion and can handle larger power output than metallic diaphragms.

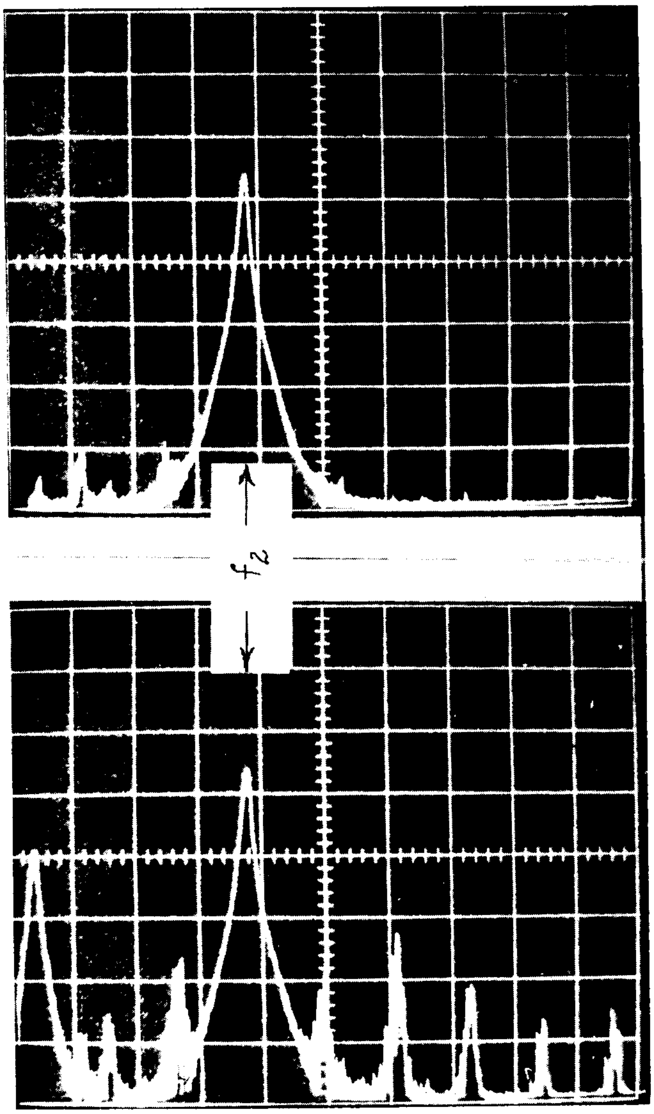

Fig. 1. Bass loudspeaker spectra. **Top**: high-quality horn-type unit at 100 dB SPL output at 2 ft. Total modulation distortion is obscured by ambient noise but may be on the order of 0.7%. **Bottom**: Ten-inch direct radiator in 1.5 ft^3 box (total enclosure) at 90 dB SPL output at 2 ft. (Vertical gain of analyzer changed to give same indicated level.) Total rms modulation distortion approximately 7%. f_1 = 50 Hz, f_2 = 300 Hz; vertical scale = 10 dB per div., horizontal scale = approximately 50 Hz per div. Note that f_1 does not show.

Upper Treble Loudspeakers

Horn type treble loudspeakers* display moderately high efficiency and the rule (which begins to look like a law) remains in force that the higher the efficiency the lower the distortion.

One "exotic" treble loudspeaker was tested. This was one of the ionized-air tweeters and it displayed over 30% rms amplitude of sum-and-difference sidebands when frequencies of 5000 Hz and 12500 Hz were mixed at 1 V input each. Listening tests showed an extreme distortion at all combinations of frequencies and at any amplitude high enough to read on a rectifier voltmeter. In fact, the "silky" highs of yesteryear's reports appear to be more "sizzly". This quality has fooled a lot of highly qualified listeners. It was surmised *a priori* that the distortion would be high. The "diaphragm" consists of the boundary between hot and cold air, and the ionized spot seems to be less than 0.1 in. in diameter. A natural conclusion would be that the modulation distortion would be of the order of 100 times as great as for a tweeter with a 1 in. diaphragm. The spectrum analyzer confirms this.

Well-designed horn tweeters with dynamic drive systems exhibit lower total modulation distortion than any other types tested here.

DEMONSTRATION TAPE

A tape was prepared of 50 Hz and 300 Hz tones mixed and played back on a small direct-radiator loudspeaker. The flutter was plainly audible.

Mozart's Concerto for Clarinet and Orchestra, third movement, contains sustained clarinet tones which tend to show even a small amount of flutter to an irritating degree. A small speaker which could execute ¼ in. of excursion at 28 Hz with almost no audible output was used to play a tape of the Concerto, while a 28 Hz oscillator tone was intermittently fed to the speaker. The resulting flutter was extremely irritating even though the modulating frequency was inaudible.

DISCUSSION

High Sound Pressures in the Throat of a Horn

Thuras, Jenkins and O'Neil [8] derive the ratio of second harmonic distortion to fundamental output of a horn

* Treble speakers are commonly referred to as "tweeters".

speaker at intense power output levels (10 W at 2000 Hz, for example) and particularly for intense pressure levels in the throat of the horn.

Whenever harmonic distortion occurs, modulation distortion likewise occurs.

This paper also mentions sum and difference frequencies. Using a horn of 200 Hz and inputs of 600 and 940 Hz, the authors detected sideband frequencies of 340 and 1540 Hz as well as the input fundamentals and harmonics. They stated that at higher power outputs the sound was very disagreeable and the fundamental tones could hardly be distinguished. Their analysis and measurements were concerned only with the harmonic distortion, and numerical data were not given for modulation distortion.

Goldstein and McLachlan [9] derive the harmonic distortion due to high pressures in the throat of a horn, but do not derive nor measure the modulation sidebands resulting from reproducing two frequencies simultaneously. They cite another paper [10] but call attention to an error of a factor of 4 in that paper.

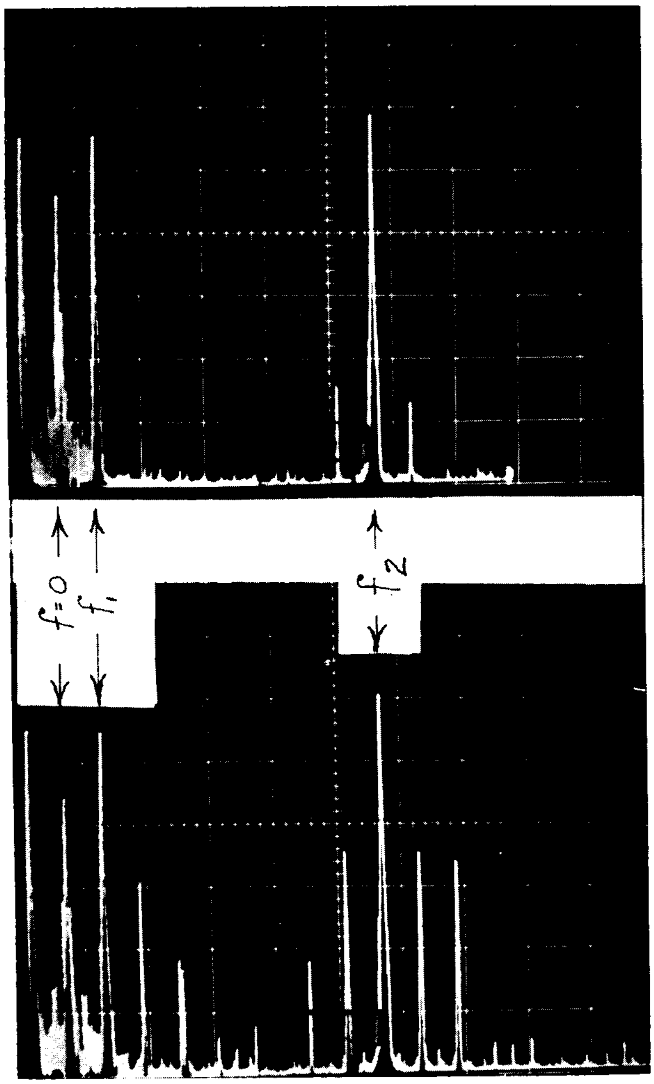

Fig. 2. Midrange loudspeaker spectra. **Top**: high-quality horn-type unit. Note absence of harmonics of the 510 Hz signal and sidebands of first-order only of 43 and 46 dB down (0.7% and 0.6%). Total rms modulation distortion, 0.9%. Output, 100 dB SPL at 2 ft. **Bottom**: typical 8 in. cone midrange loudspeaker of direct-radiator type. Note harmonics of f_1 at 23 and 35 dB down; modulation sidebands of first order at —25 dB (5.6%) and second-order sidebands of —26 dB (4.5% and —42 dB (.8%). Total rms modulation distortion, 9.1%. Output, 90 dB SPL at 2 ft. f_1 = 510 Hz, f_2 = 4.4 KHz; vertical scale = 10 dB per div., horizontal scale = approximately 500 Hz per div.

It appears obvious that second-order harmonic distortion and corresponding modulation distortion must occur in horn throats at "intense" sound pressure levels. The tests reported in the present paper involved output pressures in excess of what would be tolerable from direct radiators, but still with measured total modulation distortion on the order of ⅓ as great. Again, obviously, any device can be driven to a power level where severe distortion or failure must occur. The aim here was to deal with realistic power output levels when comparing the speaker types.

Impedance Variations with Horn Mouth Size

Wente and Thuras [11] used a bass loudspeaker with about 25 sq. ft of mouth and obtained about a 7:1 ratio of acoustic impedance. The bass horn used in present experiments had a mouth opening of 5.3 sq. ft, which with the mirror images formed by the walls in a corner produces an effective mouth area of 21 sq. ft. The motional impedance varies from 5 to 35 electrical ohm (7.5 to 38 voice coil ohm) measured over the 30 to 100 Hz range. Horns with very small mouths may exhibit acoustic impedance variations of 50:1 or more, and severe sound pressure response curve peaks and dips. Choosing a frequency at a deep response dip and forcing the input to achieve a given output could give rise to severe distortion. Even with such an inferior horn loudspeaker, the efficiency in a response dip is usually higher than for a direct radiator having the same effective area, and the distortion is lower.

There are exceptions; the reflexed midrange horn exhibited a 24 dB peak-trough difference in the response curve, and exhibited higher modulation distortion when one of the frequencies was in the bottom of the dip and the power was raised to give 100 dB SPL output. It was still, however, as good as the direct radiator. This is not a criticism of the horn, but of reflexing in such a way as to produce a violent anomaly in the response curve.

CONCLUSION

How much modulation distortion can barely be detected, how much is irritating, and how much is intolerable will vary with individual listeners, and will likely vary with any one individual as his habituation changes. What was "perfect sound reproduction" in 1905, 1925 and 1945 and what was 1955's major breakthrough is looked upon with more or less tolerance now. Our hearing has been educated to better things. In the demonstration tape, levels were chosen such that the results were unmistakable. Smaller amounts of distortion, even at and below the levels detectible by A-B comparison, may be objectionable by contributing to "listener fatigue". As people become more aware of loudspeaker faults it is reasonable to expect them to become more critical. James Moir [7] suggests that modulation distortion as low as 0.001% is detectable by ear. He does not indicate whether this numerical value is frequency deviation or sideband amplitude.

Modulation distortion cannot be eliminated in loud-

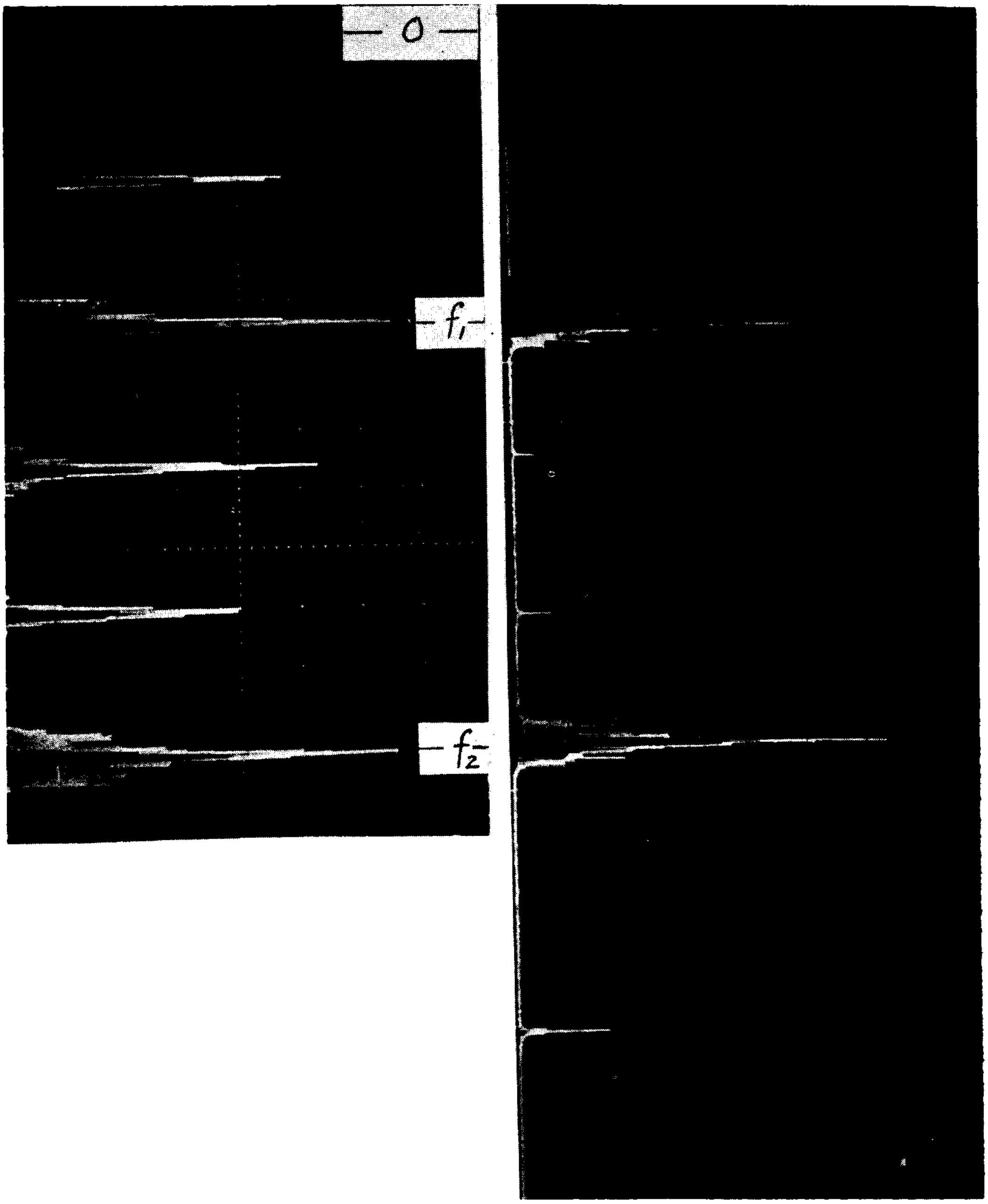

Fig. 3. Detailed spectra of high-frequency loudspeakers. **Top**: high-quality horn-type unit, paste-up of three overlapping ranges. **Bottom**: Ionized-air high-frequency loudspeaker, paste-up of two overlapping ranges. Note that the sidebands for the horn-type loudspeaker are 50 and 45 dB down from the amplitude of f_2, compared to 10 dB down (30%) distortion) for the ionized-air loudspeaker. f_1 = 5000 Hz, f_2 = 12,500 Hz at 0.5 V; vertical scale = 10 dB per div., horizontal scale = approximately 1 KHz per div.

speakers; the best that can be hoped for is minimization. If a diaphragm moves at all, it produces distortion. High-efficiency horn loudspeakers display much lower modulation distortion than the best direct radiators tested so far, and the rule seems to approach being a law that the higher the efficiency, the lower the distortion.

Among means to reduce distortion, one of the most obvious would appear to be to increase the diaphragm area. But the weight required to achieve rigidity, or the lack of rigidity, present other and more formidable problems. All large diaphragm speakers tested here exhibited audible "flexural" or "flapping" sounds.

Increasing the number of smaller direct-radiator loudspeakers has also been used. This also improves efficiency, although not to the extent realizable with well designed horns; however, the bulk and cost equal or exceed that of horns, and difficulties with polar response arise. Those observed here and elsewhere appear to have "muffled" sound, but whether this is due to the curtain of modulation distortion or to the curtain of a masking effect of enhanced bass has not been determined.

One further method of reducing the mud index in direct radiator loudspeakers is to taper the bass below about 70 Hz. Since little or no fundamental sound is radiated by a symphony orchestra below about 45 Hz, tapering would entail a loss of, say, 5 dB at the lowest

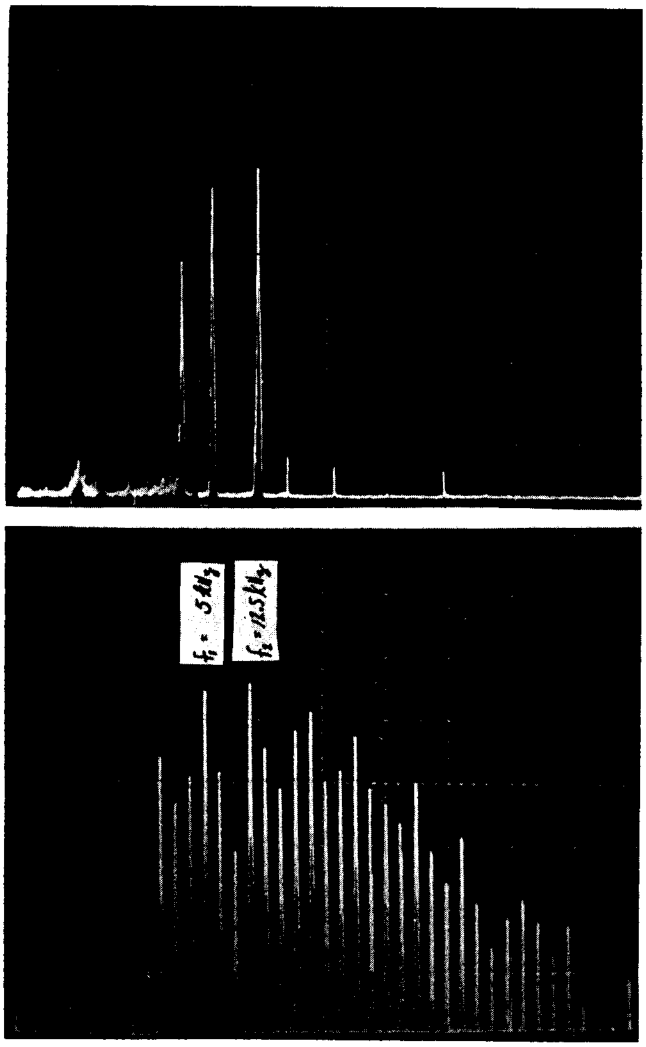

Fig. 4. Far spectra of the high-frequency loudspeakers of Fig. 3. **Top**: high-quality horn-type loudspeaker. **Bottom**: ionized-air loudspeaker. The B&K ½ in. microphone purports to be substantially flat to 40 KHz. Note that the ionized-air loudspeaker may be generating sidebands at much higher frequencies than shown; those in the audible range amount to 30% rms of the amplitude of f_2. $f_1 = 5000$ Hz, $f_2 = 12{,}500$ Hz at 0.5 V; vertical scale = 10 dB per div., horizontal scale = 10 KHz per div.

fundamentals radiated by a large orchestra, and modulation distortion would be reduced by more than 5 dB.

Apparently horn loading remains the best means to reduce diaphragm excursion and increase efficiency so as to minimize modulation distortion.

POSTLUDE

It was hoped early in these experiments that amplitude and frequency modulation could be distinguished, but so far this separation has been elusive. Pragmatically, the total modulation distortion is what affects quality, and when horn-type loudspeakers are shown to be capable of higher output at lower total modulation distortion, it seems fair to conclude that the sum of the amplitude and frequency modulation distortion is really the sought quantity.

However desirable it would be to measure each separately, this remains "beyond the scope of this paper".

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Modulation Distortion in Loudspeakers: Part II

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Modulation distortion in loudspeakers consists of amplitude modulation distortion (AMD) and frequency modulation distortion (FMD); the effective sum of these is the total modulation distortion (TMD). It appears that the first-order side frequencies are due mainly to frequency modulation and the second-order sideband frequencies are due to amplitude modulation. Small direct-radiator loudspeakers typically display large AMD and relatively less FMD, while horn loudspeakers display small FMD and negligible AMD.

INTRODUCTION Part I of this paper [1] defined the various kinds of distortion. The previous paper was concerned mainly with frequency modulation distortion (FMD), which appeared to be of greater importance than amplitude modulation distortion (AMD). Applying the spectrum analyzer to small direct radiators shows that AMD may exceed FMD by an order of magnitude.

ANALYSIS

A former colleague suggested that in a symmetrical system first-order sideband frequencies would not exist [2].
Let

$$y = k(x - mx^3) \tag{1}$$

and assume this to be a reasonable approximation to the displacement y of a loudspeaker diaphragm for an applied force of x, to be considered valid over the range of

$$-2.0 < x < +2.0$$

Figure 1 illustrates this "stress–strain diagram", where, for example, $m = 0.1$, or

$$y = k(x - 0.1x^3). \tag{2}$$

Now assume two equal sinewaves of unit amplitude,

$$x = \sin \omega_1 t + \sin \omega_2 t \tag{3}$$

$$\begin{aligned} y = {} & \sin \omega_1 t + \sin \omega_2 t \\ & -0.1\,[\sin^3 \omega_1 t + 3 \sin^2 \omega_1 t \sin \omega_2 t \\ & + 3 \sin \omega_1 t \sin^2 \omega_2 t \\ & + \sin^3 \omega_2 t]. \end{aligned} \tag{4}$$

Using $\sin^3 a = 3/4 \sin a - 1/4 \sin 3a$; $3 \sin^2 a \sin b = 3\,(1/2 - 1/2 \cos 2a) \sin b + 3\,(1/2 - 1/2 \cos 2b) \sin a = 3/2 \sin b - 3/2 \sin (a + 2b) - 3/2 \sin (a - 2b)$, etc., we obtain

$$\begin{aligned} y = {} & \sin \omega_1 t + \sin \omega_2 t \\ & -0.1\,[3/4 \sin \omega_1 t - 1/4 \sin 3\omega_1 t \\ & + 3/4 \sin \omega_2 t - 1/4 \sin 3\omega_2 t \\ & + 3/2 \sin \omega_1 t + 3/2 \sin \omega_2 t \\ & - 3/2 \sin (\omega_1 t + 2\omega_2 t) \\ & - 3/2 \sin (\omega_1 t - 2\omega_2 t) \\ & - 3/2 \sin (\omega_2 t + 2\omega_1 t) \\ & - 3/2 \sin (\omega_2 t - 2\omega_1 t)], \end{aligned} \tag{5}$$

or

$$\begin{aligned} y = {} & 0.775 \sin \omega_1 t + 0.775 \sin \omega_2 t \\ & + 0.025 (\sin 3\omega_1 t + \sin 3\omega_2 t) \\ & - 0.15\,[\sin (\omega_1 t + 2\omega_2 t) \\ & + \sin (\omega_1 t - 2\omega_2 t) \\ & + \sin (\omega_2 t + 2\omega_1 t) \\ & + \sin (\omega_2 t - 2\omega_1 t)]. \end{aligned} \tag{6}$$

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second-order sidebands of f_2 are below the resolution of the spectrum analyzer, it is to be assumed that amplitude modulation is negligible and that the sidebands are due to FMD.

The lower curve in Fig. 2 shows the spectrogram of the 8 in. direct radiator. As in the top figure, the first peak is the amplitude of f_1, but followed by small amounts of second and third harmonic distortion. Next is a large peak, amplitude of f_2, flanked by first- and second-order sideband amplitudes. Then comes the second harmonic $2f_2$ flanked by its sidebands of $2f_2 \pm f_1$, and finally $3f_2$. All the components predicted by Eq. (6) are represented. The magnitude of the $2f_2$ component is much larger than would be expected from Eq. (6) and remains unexplained, except that direct-radiator loudspeakers do unexplained things. Perhaps a nodal cone breakup was taking place and the microphone was in just the right place to maximize the fault. The fact that the second harmonic of f_1 predominates over the third harmonic suggests that the first-order sideband amplitudes of f_2 contain both AMD and FMD.

A significant observation is that the direct-radiator midrange loudspeaker had to be driven into a nonlinear range of cone travel to produce 100 dB SPL at 2 ft. This level corresponds to about 90 dB at a normal listening distance in a typical listening room. This is 1/100 the peak sound power one would demand for "realistic music reproduction", but one sees various orders of modulation distortion in amounts up to 15%. By contrast, the high-quality horn loudspeaker shows a mere 1% total modulation distortion at the same output power.

Another significant observation is that the high-quality horn displays only first-order modulation distortion, which is probably the irreducible frequency modulation type. Again, by contrast, the direct radiator shows a much higher level of first-order sideband components, suggesting suspension asymmetry plus higher FMD output.

TEST OF A FULL-RANGE LOUDSPEAKER

The speaker chosen for this test was a direct radiator consisting of several small cone loudspeakers of "long throw" capability and with a total area approximating that of a 12 in. cone loudspeaker. This system was intended for "full frequency range" and normally employed with an equalizer. The two frequencies were f_1 = 50 Hz and f_2 = 750 Hz, both adjusted to produce 95 dB SPL at 2 ft.

Figure 3 shows the spectrogram of this test. The first peak is the amplitude of the output of f_1. Following this, barely discernible, is the second harmonic ($2f_1$), followed by a strong $3f_1$ (about 20 dB down or 10% third harmonic). Fourth and fifth harmonics are significant.

The next major peak is f_2 (same amplitude as f_1, 95 dB) flanked by small (−30 dB) first-order sidebands ($f_2 \pm f_1$), in turn are flanked by larger second-order sidebands ($f_2 \pm 2f_1$). Still higher-order sidebands are visible.

From an analysis such as Eq. (6), and from the earlier paper, it would appear that there is about 3% FMD and 14% AMD. Apparently the FMD is not as serious as the AMD for this particular loudspeaker.

Sidebands of order higher than the second are not explained by Eq. (6); however, the derivation assumed symmetry, and some asymmetry existed as indicated by presence of a second harmonic of f_1.

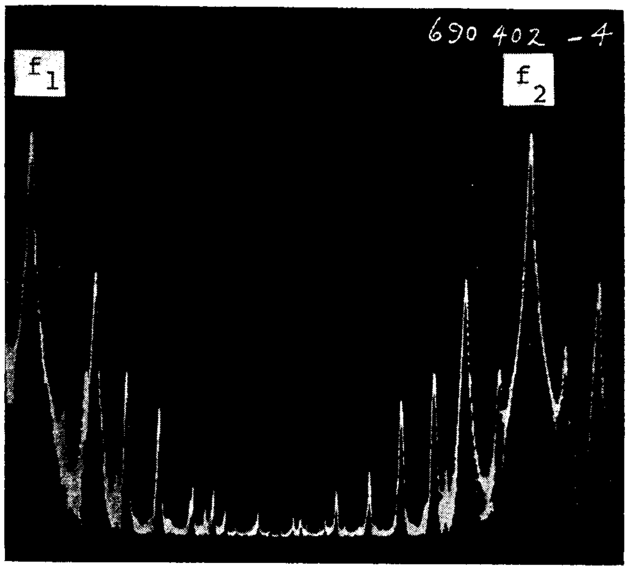

Fig. 3. Spectrogram of small full-range direct radiator employing several small cone loudspeaker elements and a preamplifier equalizer. First peak f_1 = 50 Hz, followed by a small $2f_1$ component, and $3f_1$, $4f_1$, etc. Second large peak, f_2 = 750 Hz, flanked by small first-order sidebands of $f_2 \pm f_1$ and larger sidebands of $f_2 \pm 2f_1$, and significant third- and fourth-order sideband frequencies. Total modulation distortion, approximately 14%. Input of both frequencies adjusted to produce 95 dB SPL at 2 ft.

It is easy to ignore sideband amplitudes of less than 3% when there are distortion amplitudes exceeding 10%. It would be interesting to find the causes of these unpredicted distortion products, but the cone loudspeaker with its infinite number of modes of vibration and breakup could take a lifetime of studying third order effects.

DISCUSSION

Beers and Belar [3] suggested using different speakers for bass and treble. An examination of Fig. 2 suggests that this expedient does not go far enough. Here is an example of a direct-radiator midrange loudspeaker such as is used in a three-way system, producing excessive distortion within its own normal band. The horn-loaded loudspeaker displays about 1/10 the distortion of the direct radiator.

In the case of the multiple loudspeaker whose performance is shown in Fig. 3, obviously the mere proliferation of the number of loudspeakers fails to reduce distortion to tolerable levels. In the companion paper a horn woofer was tested at 100 dB SPL and found to produce less than 1% total modulation distortion.

The frequency response curves of the three loudspeakers tested are shown in Figs. 4 and 5.

The solid curve in Fig. 4 shows the frequency response of the horn midrange, the distortion of which is shown in Fig. 2a. The partly dashed curve in Fig. 4 is for the direct radiator depicted in Fig. 2b.

Figure 5 shows the response (including equalizer) of the loudspeaker whose distortion is depicted in Fig. 3.

One should not expect a correlation between distortion

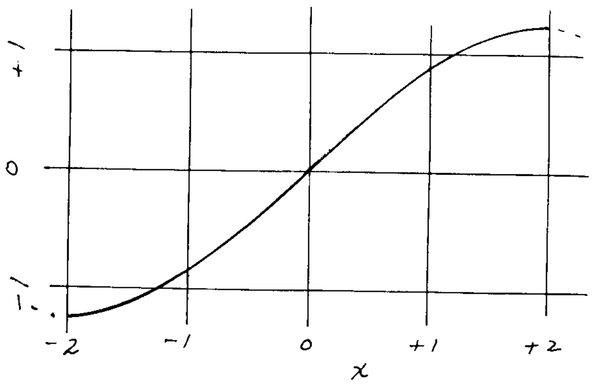

Fig. 1. Simplified stress–strain (force *vs* displacement) curve for suspended system of a loudspeaker, assuming symmetry of both magnetic and compliance systems.

The bracketed term in Eq. (6) shows modulation sideband frequencies of $f_1 \pm 2f_2$ and $f_2 \pm 2f_1$.

Note that these are second-order sidebands; there are no first-order sidebands of the form $f_2 \pm f_1$.

One may conclude that in a symmetrical system, such as illustrated in Fig. 1, only second-order amplitude-distortion sideband frequencies exist.

In Part I of this paper [1], an example was used wherein the amplitudes of the frequency modulation sideband frequencies were 0.017 (1.7%) for the first order and 0.00013 for the second order. These second-order amplitudes are 40 dB down lower than the first-order ones and not apt to show on an analyzer spectrogram.

Thus it appears that first-order sideband amplitudes can be attributed mainly to frequency modulation distortion and second-order sideband amplitude entirely to amplitude modulation distortion.

Of course, asymmetric nonlinearity would induce some first-order AMD, but it would seem that any second-order sideband frequencies must be almost entirely due to AMD.

INTERPRETATION

Examining Eq. (6) further, the fundamental signals, f_1 and f_2, which started out at unity are reduced to 0.775. There is a 2.5% third harmonic of each input signal. These effects are intuitively obvious, at least qualitatively. The amplitude of each sideband frequency which reads 0.15 amplitude becomes approximately 19% of the 0.775 fundamental output.

The bracketed terms show amplitudes of frequencies $f_2 \pm 2f_1$. This indicates that the symmetrical stress–strain systems depicted in Fig. 1 give rise to second-order sidebands, with complete absence of first-order sideband frequencies of $f_2 \pm f_1$.

The bracketed terms show amplitudes of frequencies of $f_1 \pm 2f_2$. Recognizing that $\sin(-a) = -\sin a$, it is logical to conclude that, with appropriate phase shifts, these sideband terms may be written as amplitudes of $2f_2 \pm f_1$.

This turns out to be a surprise. Early work with the sprectrum analyzer did not show these high-order terms for the simple reason they were not suspected and the "window" of the analyzer was not wide enough to include them. Therefore, in some of the new study the analyzer was adjusted to "see" out past $3f_2$.

TESTS OF MIDRANGE LOUDSPEAKERS

Two midrange loudspeakers were compared. One was a horn-loaded system designed for the 400 to 6000 Hz range, the other was an 8 in. direct-radiating cone designated by its manufacturer specifically for midrange application. Frequencies of f_1 = 540 Hz and f_2 = 4400 Hz were used, and inputs adjusted to give outputs of 100 dB SPL at 2 ft for f_1 and 92 dB for f_2. The vertical scale is 10 dB per major division.

Figure 2 shows the two spectrograms. The upper curve depicts the performance of the horn loudspeaker: the first peak is the amplitude of f_1, and no harmonics of f_1 are seen. The next large peak is the amplitude of f_2, flanked by two small first-order sideband amplitudes. The sideband amplitudes are nearly 40 dB below the amplitude of f_2, so that the total modulation distortion is slightly over 1%.

Since harmonic distortion of f_1 is not visible and

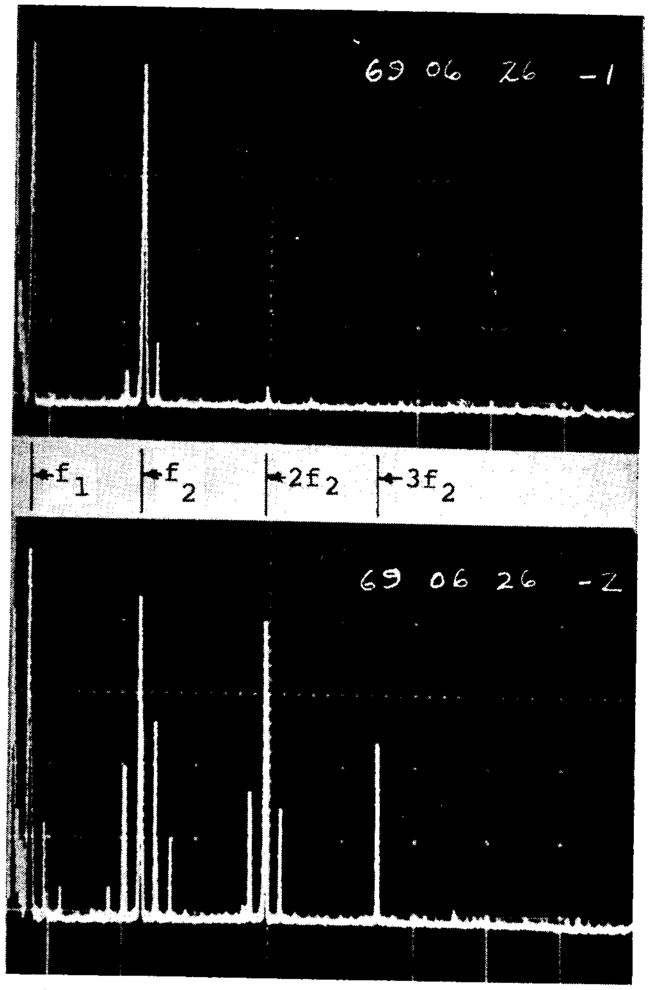

Fig. 2. Spectrograms of *a*. horn and *b*. direct-radiator midrange loudspeakers. Output 100 dB SPL at 2 ft for f_1 = 540 Hz, 92 dB for f_2 = 4400 Hz. Vertical scale 10 dB per major division. First two major peaks are f_1 and f_2. In top figure, the only significant distortion showing is the pair of sideband frequencies $f_2 \pm f_1$. In the bottom figure significant distortion components are $2f_1$, $f_2 \pm f_1$, $f_2 \pm 2f_1$, $2f_2$, $2f_2 \pm f_1$, and $3f_1$. Total distortion over 10%.

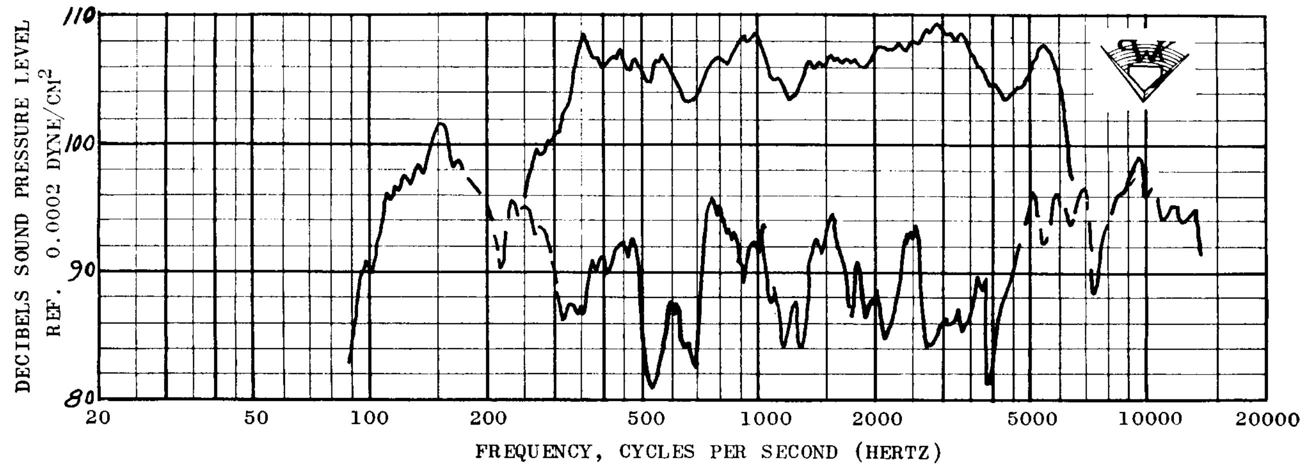

Fig. 4. Response curves of the midrange loudspeakers with distortion shown in Fig. 2. *Top curve.* Horn system; *Bottom curve,* 8 in. direct radiator, both at 1 W input.

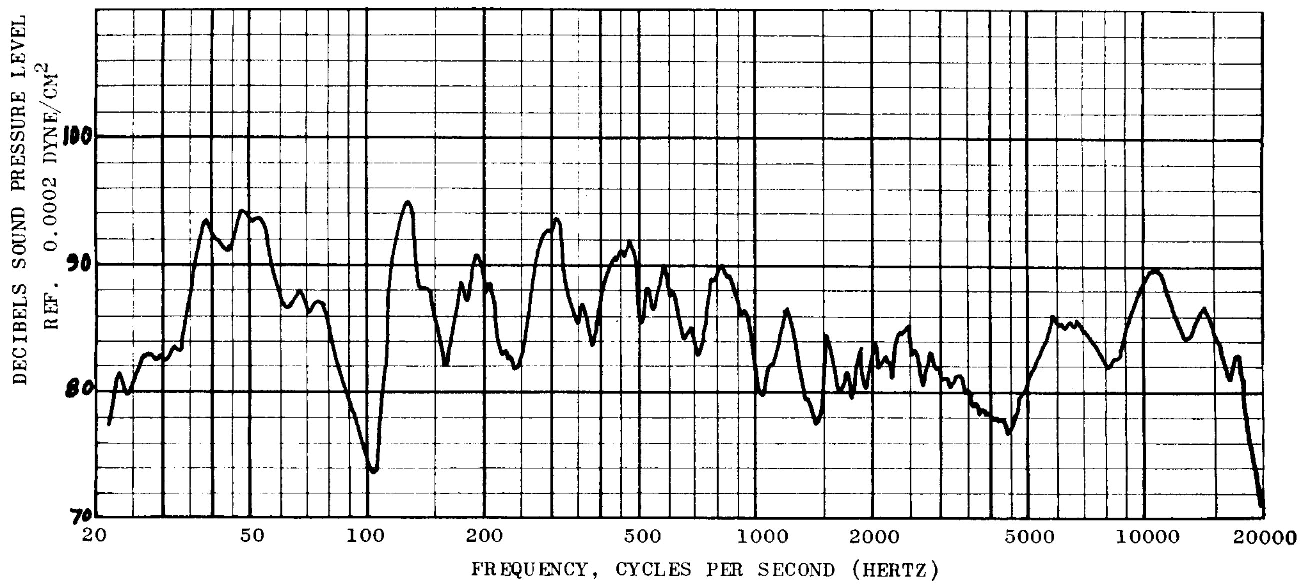

Fig. 5. Frequency response of the loudspeaker with distortion shown in Fig. 3. Input 0.5 W at 500 Hz, approximately 50 W at 40 Hz (difference due to equalizer); curve level coordinates corrected to 1 W input at 500 Hz.

of a loudspeaker and its frequency response, but in these cases it appears the response peak–trough ratio does correlate with the amount of total modulation distortion.

CONCLUSION

It is believed that Figs. 2 and 3 illustrate at least qualitatively what can be concluded from Eq. (6) and from Part I of this paper. The separation of FMD and AMD may not be absolute and precise, but it appears that first-order sideband components must be largely FMD and second-order components largely AMD. Further, it seems to follow that high-efficiency horns will display small FMD and negligible AMD compared to direct radiators, in which both forms of distortion are higher. Small direct radiators driven to output levels necessary for "realistic reproduction of music" may display a preponderance of AMD over FMD, and at objectionably high distortion levels.

A technical conclusion is that, to a reasonable approximation, the spectrum analyzer shows first-order sideband frequency components as FM distortion and second-order components as AM distortion.

A practical conclusion is that the inherently low distortion of properly designed horn-type loudspeakers is significant in the virtual elimination of amplitude modulation distortion and the reduction of frequency modulation distortion to nearly irreducible limits, and that this low distortion is the main contribution to the "cleanness" of sound reproduction from loudspeakers of this type.

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Modulation Distortion in Loudspeakers: Part III*

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Distortion in loudspeakers is shown to be nearly proportional to power output. Typically a plot of log distortion versus dB output shows a 1:1 relation. In one sample loudspeaker the slope of the distortion versus output curve was in excess of 45 degrees. Comparison is shown between a direct radiator of 20-cm (8-in) diameter, one of 30-cm (12-in) diameter, and a high-efficiency horn of 0.45 m³ (16 ft³). At 95-dB sound pressure level output measured at 61 cm (2 ft) the 12-cm cone showed 18% (—15 dB ref 100%), the 30-cm cone showed 6% (—25 dB ref 100%), and the horn showed 0.8% (—42 dB ref 100%). Each curve of distortion versus output shows a slope of at least 45 degrees.

INTRODUCTION: In the popular "hi fi" press it has been stated that distortion in loudspeakers increases only slightly with power. Intuition or common sense dictate that this is a fallacy. Since intuition and especially common sense are rare in the "high fidelity" art, tests were conducted to support the intuitive logic that dictates distortion to be proportional to loudspeaker power output.

EXPERIMENTAL PROCEDURE

The spectrum analyzer was employed to determine harmonic distortion at different output levels and the modulation distortion resulting from a mixture of two frequencies. Three loudspeaker samples were used as tabulated in Table I.

Test frequencies of $f_1 = 41$ Hz and $f_2 = 350$ Hz were chosen, since in each case these frequencies would be radiated from the same bass diaphragm. The horn system crossover is 400 Hz. Of course, use of 41 and 1300 Hz would have resulted in substantially zero distortion for the horn, and very high first-order distortion for the direct radiators.

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In the case of the 30-cm speaker and the horn, equal sound pressure levels were used for the two frequencies. In the case of the 12-cm speaker, the amplitude of the higher frequency was 6 dB lower than the amplitude of the lower frequency.

In each case, harmonic distortion of f_1 varied only slightly with presence or absence of f_2, and in all cases the harmonics of f_1 were smaller than the sideband amplitudes of $f_2 \pm f_1$ and $f_2 \pm 2f_1$.

Since harmonic distortion contributed negligibly to the total distortion, and it was desired to plot a single value of distortion versus power, the total rms value of all significant sideband amplitudes was computed and plotted in dB.

Table I.

Sample Number	1 Direct Radiator	2 Direct Radiator	3 Large Horn
Basket diameter	30.5 cm (12 in)	20 cm (8 in)	
"Effective" cone diameter	23 cm (9 in)	16 cm (6.5 in)	
"Rigid" cone diameter	20 cm (8 in)	15 cm (6 in)	
Total bulk (approximately)	40 000 cm³ (1.5 ft³)	30 000 cm³ (1 ft³)	0.45 m³

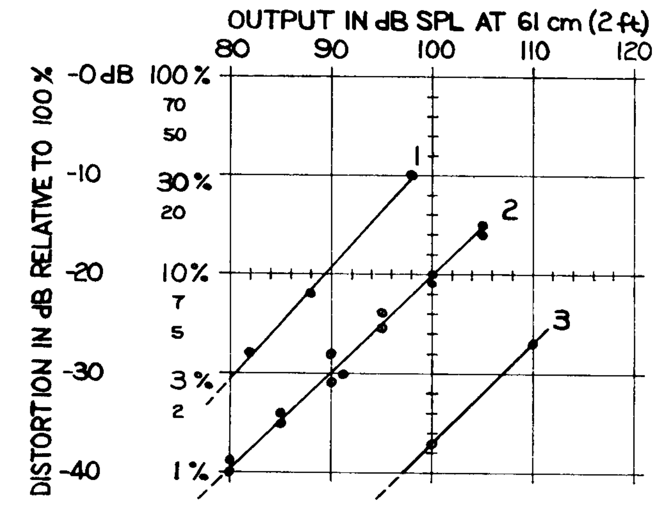

Fig. 1. Distortion versus output for 3 loudspeakers. Total distortion is mainly modulation (IM) resulting from mixture of $f_1 = 41$ Hz, $f_2 = 350$ Hz; harmonic distortion was from 7 to 10 dB lower than IM distortion in all cases. Curve 1—20-cm (8-in) direct-radiator loudspeaker in 30 000-cm³ box (1 ft³); peak power input at 98-dB output was 57 watts (limit of loudspeaker); curve 2—30-cm (12-in) direct-radiator loudspeaker in 40 000-cm³ box (1.5 ft³); peak power input at 105-dB output was 179 watts (limit of amplifier); curve 3—large-horn woofer, total bulk 0.45 m³ (16 ft³); peak power input at 110-dB output was 13 watts; no attempt was made to reach limits of either loudspeaker or amplifier.

Fig. 1 gives the rms distortion in dB (referred to 100% of the amplitude of the modulated frequency), plotted versus power output measured at 61 cm (2 ft).

RELATIVE IMPORTANCE OF AM, FM, AND HARMONIC DISTORTION

There are those who contend that Doppler (frequency modulation) distortion is inconsequential, the reasoning being based on the small frequency deviation which occurs. Using the example of a diaphragm excursion of 6 mm (¼ in) at 50 Hz, the displacement would be

$$x = \frac{1}{4} \sin (2\pi \times 50t)$$

and the velocity

$$v = \frac{1}{4} \times 2\pi \times 50 \cos (2\pi \times 50t)$$

$$v_{peak} = 200 \text{ cm/s } (79 \text{ in/s}).$$

Since the velocity of sound is $c = 344$ m/sec (13 500 in/s), the frequency deviation of a higher frequency being radiated from the same diaphragm would be only

$$\Delta v/c = 79/13\,500 = 0.0058$$

or a trifle over ½%. But the amplitudes of the sideband frequencies are known to be [1]

$$d = 0.033\, A_1 f_2 \quad (\%)$$

where A_1 is the amplitude of the motion at the lower frequency f_1 taken in the example as ¼ in, f_2 is the frequency of the modulated signal, and d is the rms sum of the sideband amplitudes expressed in percent of the amplitude of the higher frequency signal. Assuming $f_2 =$ 350 Hz, then

$$d = 2.9\%$$

which may not be impressively large, but experiments show that it is plainly audible.

Then the critics of FM distortion fail to realize that AM distortion is frequently larger than FM distortion.

In the case of the 20-cm (8-in) direct-radiator cone the performance of which is depicted as curve 1 in Fig. 1, the AM component exceeded the FM component of total IM distortion. The maximum total IM distortion in the 20-cm cone was nearly 30%, of which only about 5% could be accounted for by the FM components.

Devotees of direct-radiator speakers view with alarm the high throat pressures in horns which allegedly produce high harmonic distortion [2], [3]. In the case of the horn (distortion depicted in curve 3, Fig. 1) harmonic distortion was 9 dB below the curve of total distortion.

DISCUSSION

Note that 80-dB output, and 1% distortion or 40 dB below 100% together represent a distortion output of 40-dB sound pressure level. The noise level in the test room, unweighted, was nearly 50 dB. If the analyzer had not been highly selective, measurements could not have been made down to 40 dB. As it was, determinations below −40 dB ref 100% entailed careful examination of the spectrograms to distinguish between noise and an almost vague glitch identifiably due to a sideband amplitude. It is submitted that the reviewers who felt that distortion increased only slightly with power were working at too low a level and were probably reading the output on a meter instead of a scope so that distortion and noise were indistinguishable. Obviously below the noise level the distortion plus noise will be constant and the ratio of distortion plus noise would first flatten and then assume a negative slope. While this is expressed as an opinion, it seems to be supportable.

CONCLUSION

The fact of the case is that distortion is closely proportional to power output. Also the distortion is closely proportional to diaphragm excursion. The 30-cm direct-radiator cone had to move about ⅓ as far as the 20-cm direct-radiator cone for the same output, and the distortion was close to 10 dB lower. The horn diaphragm motion was too small to be measured but could be estimated to be about ⅛ that of the 30-cm cone, so the 17-dB difference in distortion is the right order of magnitude.

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Note: Mr. Klipsch's biography appeared in the October 1972 issue.